Exercise 12

Check the validity of the maximum principle for the harmonic function $(1-x^2-y^2)/(1-2x+x^2+y^2)$ in the disk $\overline{D} = \{x^2+y^2 \leq 1\}$. Explain.

Solution

Let the harmonic function be denoted by $u(x,y)$.

$$\begin{aligned} u(x,y) &= \frac{1-x^2-y^2}{1-2x+x^2+y^2} \\ &= \frac{1-x^2-y^2}{(1-x)^2+y^2} \end{aligned}$$

The maximum principle does not apply to this harmonic function because $u(x,y)$ is not continuous at one of the points in $\overline{D}$, that is, the limit of $u(x,y)$ as $(x,y) \to (1,0)$ does not exist.